## PACE INSTITUTE OF TECHNOLOGY \& SCIENCES::ONGOLE (AUTONOMOUS)

II B.TECH I SEMESTER END REGULAR EXAMINATIONS, JAN - 2023
TRANSFORMATION TECHNIQUES \& PARTIAL DIFFERENTIATION (Common to EEE,ME,ECE,IT,CSE(IOTCSBT),AIDS,AIML Branches)
Time: 3 hours
Max. Marks: 70
Answer all the questions from each UNIT (5X14=70M)

| Q.No. |  | Questions | Marks | CO | KL |
| :---: | :---: | :---: | :---: | :---: | :---: |
| UNIT-I |  |  |  |  |  |
| 1 | a) | Fint ${ }^{\text {a }}$ | [7M] | 1 | 1 |
|  |  | Find the Fourier Series of $\mathrm{f}(\mathrm{x})=2 \quad 2 \quad$ in $0 \leq x \leq 2 \pi$. |  |  |  |
|  | b) | Find the Fourier Series of $\mathrm{f}(\mathrm{x})=e^{-x} e^{-x}$ in $-1 \leq \mathrm{x} \leq 1$. | [7M] | 1 | 1 |
| OR |  |  |  |  |  |
| 2. | a) | Find the Fourier Series of $\mathrm{f}(\mathrm{x})=\|x\|\|x\|$ in $-\pi \leq \mathrm{x} \leq \pi$ and hence deduce that $\frac{1}{1^{2}}+\frac{1}{3^{2}}+\frac{1}{5^{2}}+----=\frac{\pi^{2} 1}{81^{2}}+\frac{1}{3^{2}}+\frac{1}{5^{2}}+----=\frac{\pi^{2}}{8}$ | [7M] | 1 | 1 |
|  | b) | Find the half range cosine series of $\mathrm{f}(\mathrm{x})=\mathrm{x}$ in $0<\mathrm{x}<1$. | [7M] | 1 | 1 |
| UNIT-II |  |  |  |  |  |
| 3. | a) | Using Fourier Integral, show that $e^{-x} \cos x=\frac{2}{\pi} \int_{0}^{\infty} \frac{\lambda^{2}+2}{\lambda^{4}+4} \cos \lambda x d \lambda$ | [7M] | 2 | 3 |
|  | b) | $\begin{aligned} & e^{-x} \cos x=\frac{2}{-} \int_{-}^{\infty} \frac{\lambda^{2}+2}{-} \cos \lambda x d \lambda \text { form of } \mathrm{f}(\mathrm{x}) \text {, then the complex Fourier } \\ & \text { transform of } f(x) \cos a x \text { is } \frac{1}{2}[F(p+a)+F(p-a)] \end{aligned}$ | [7M] | 2 | 3 |
| $f(x) \cos a x$ is $\frac{1}{-}[F(p+a)+F(p-a)]$ |  |  |  |  |  |
| 4. | a) | Express $f(x)=\left\{\begin{array}{c}1 \text { in } 0 \leq x \leq \pi \\ 0 \text { in } x>\pi\end{array}\right.$ f $\left.x\right)=\left\{\begin{array}{c}1 \text { in } 0 \leq x \leq \pi \\ 0 \text { in } x>\pi\end{array}\right.$ as a Fourier Sine Integral and hence evaluate $\int_{0}^{\infty} \frac{1-\operatorname{Cos}(\pi \lambda)}{\lambda} \operatorname{Sin}(\mathrm{x} \lambda) \mathrm{d} \lambda$. | [7M] | 2 | 5 |
|  | b) | $\int_{0}^{\infty} \frac{1-\operatorname{Cos}(\pi \lambda)}{\lambda} \operatorname{Sin}(\mathrm{x} \lambda) \mathrm{d} \lambda$. Cosine Transforms of x . | [7M] | 2 | 1 |
| $\lambda$ UNIT-III |  |  |  |  |  |
| 5. | a) | $\text { Prove that } z\left[n^{2}\right]=\frac{z^{2}+z}{(z-1)^{z}} z\left[n^{2}\right]=\frac{z^{2}+z}{(z-1)^{3}} .$ | [7M] | 3 | 5 |
|  | b) | Find the $Z$ transform of $\cosh n \theta$. | [7M] | 3 | 1 |
| OR |  |  |  |  |  |
| 6. | a) | Using Convolution theorem, find $Z^{-1}\left[\frac{z^{3}}{Z(z-2)(z-3)}\right]$ | [7M] | 3 | 3 |
|  | b) | Solve the difference equation $u_{n+2}+4 u_{n+1}+3 u_{n}=3^{n}$ with $u_{0}=0, u_{1}=1$ | [7M] | 3 | 3 |
| $u_{n+2}+4 u_{n+1}+3 u_{n}=3^{n} \text { with } u_{0}=0, u_{1}=1$ |  |  |  |  |  |
| 7. | a) | $\text { Verify Euler's theorem for } u=x^{2} \tan ^{-1}\left(\frac{y}{x}\right)-y^{2} \tan ^{-1}\left(\frac{x}{y}\right)$ | [7M] | 4 | 3 |
|  | b) | $\begin{aligned} & u=x^{2} \tan ^{-1}\left(\frac{y}{x}\right)-y^{2} \tan ^{-1}\left(\frac{x}{y}\right) \quad \frac{\partial(x, y)}{\partial(r, \theta)}=1 . \\ & \text { If } \quad x=e^{r} . \operatorname{Sec} \theta ; y=e^{r} . \operatorname{Tan} \theta \quad \text { then prove that } \end{aligned}$ | [7M] | 4 | 5 |
| OR |  |  |  |  |  |
| 8. | a) | Expand $\mathrm{e}^{\mathrm{xy}}$ in the neighbourhood of (1, 1). | [7M] | 4 | 2 |


|  | b) | Find the volume of the greatest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}+\frac{z^{2}}{c^{2}}=1 \frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}+\frac{z^{2}}{c^{2}}=1$. | [7M] | 4 | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| UNIT-V |  |  |  |  |  |
| 9. | a) | Form the Partial Differential Equation by eliminating $a$ and $b$ from $z=\left(x^{2}+a\right)\left(y^{2}+b\right)$. | [7M] | 5 | 3 |
|  | b) | Solve the $\mathrm{PDE} x^{2} p^{2}+y^{2} q^{2}=1 x^{2} p^{2}+y^{2} q^{2}=1$. | [7M] | 5 | 3 |
| OR |  |  |  |  |  |
| 10. | a) | Solve the partial differential equation $p x-q y=y^{2}-x^{2}$. | [7M] | 5 | 3 |
|  | b) | Solve the partial differential equation $\left(D-D^{\prime}-1\right)\left(D-D^{\prime}-2\right) z=e^{2 x-y}$ | [7M] | 5 | 3 |
| $\left(D-D^{\prime}-1\right)\left(D-D^{\prime}-2\right) z=e^{2 x-y}$ ***** |  |  |  |  |  |

